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FROM BIT-STRINGS TO QUATERNIONS*

or

FROM HERE TO ETERNITY?

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Submitted to *RECONSTRUCTION OF QUANTUM MECHANICS (**RQM**)*

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ABSTRACT

This is a preliminary version of my paper for Proc. ANPA 12. **COMMENTS ARE EAGERLY SOLICITED.** If not received in time to make it into the final version, they will still be useful for my papers at ANPA WEST 7 and ANPA 13.

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1. INTRODUCTION

At ANPA 9, 10, 11 and 12 I attempted to give rigorous mathematical justification for the *discrete physics* model of relativistic quantum scattering theory sketched out in what has become the basic published reference[1]. For various technical reasons, I failed to convince the audience that I had achieved this goal. There are deep philosophical reasons underlying the controversy. I now realize that these might have prevented acceptance of my approach even if the mathematics had turned out to be impeccable. These issues turn on the differences in objective between the West Coast "Bolsheviks" — as Clive [Kilmister] has dubbed us— and the English "Mensheviks". We believe that the philosophical position underlying *discrete physics* has been carefully spelled out by Christoffer [Gefwert][2] and — in careful coordination with that paper — by David [McGoveran][3]; we will not repeat that foundational discussion here.

Prior to ANPA 12, I circulated a technical note[4] on various salient points of disagreement to those of us who are actively engaged in trying to reconstruct quantum mechanics, and to some critics of our efforts. This provoked a position paper "of the same logical type" from Bastin and Kilmister, presented at ANPA 12 and revised for these Proceedings. So far as I am concerned, none of this technical discussion has affected any of the physical conclusions about the consequences of adopting the discrete physics approach to relativistic quantum mechanics and relativistic cosmology presented in FDP, DP and subsequent papers. Hopefully this controversy will eventually lead to a technical solution that cannot be faulted either from a mathematical perspective or from any philosophical point of view which shares our basic *modeling methodology.*

Unfortunately as I go to press this result has not been achieved. Fortunately ANPA is a place where I can present "work in progress" and expect constructive and useful criticism. I trust you will read the following incomplete development in that spirit.

In Chapter 2 I follow the route from bit-strings to quaternions as far as I can. As Stan Gudder pointed out to me last spring, to complete this work would require me to model "vector addition". I now realize that this necessarily "expands the space" in which the finite vectors with which we start operate, and hence *must* involve concatenation as well as discrimination. Thus it has more to do with cosmology than the particle physics which is my immediate focus. In thinking about this I finally realized that for particle physics (or minimally for finite particle number relativistic scattering theory) all I probably need is rotations and boosts, not vector addition. Further thought led me to try modeling quantum numbers directly with less reliance on "geometric" visualizations. I have made some progress along these lines, and report it in Chapter 3.

2. BIT-STRING COORDINATES

2.1. Greider's Quaternions

Our objective in this chapter is to map bit-strings onto quaternion coordinates which are integral, or rational. Our strategy is to construct the ingredients used by Greider[5] in his systematic development of the scalars, 4-vectors, bivectors trivectors and pseudoscalars needed in relativistic quantum field theory. We choose his approach because he has demonstrated that ambiguities in formulating the free field conservation laws using the tensor notation are uniquely resolved within his formalism; further, his approach can readily be extended to general relativity. He starts from the basic bivector product

$$\mathbf{e}_\mu \mathbf{e}_\nu + \mathbf{e}_\nu \mathbf{e}_\mu = 0; \mu \neq \nu;\ \ \mu, \nu \in 0,1,2,3 \tag{2.1}$$

and the scalar products

$$\mathbf{e_1}^2 = \mathbf{e_2}^2 = \mathbf{e_3}^2 = -\mathbf{e_0}^2 = +1 \tag{2.2}$$

He defines a 4-vector $\mathbf{v}$ by its projection onto this basis, i.e.

$$\mathbf{v} := v^0\mathbf{e_0} + v^1\mathbf{e_1} + v^2\mathbf{e_2} + v^3\mathbf{e_3} = (v^1, v^2, v^3; v^0) = (\vec{v}; v^0) \tag{2.3}$$

from which the Lorentz-invariant (*space-like* positive) 4-vector product

$$\sigma^2 := \vec{a}\cdot\vec{b} - a^0b^0 \tag{2.4}$$

follows immediately. Note Greider's arbitrary choice of a space-like metric for σ^2 rather than the *time-like* positive metric

$$\tau^2 := -\sigma^2 = a^0b^0 - \vec{a}\cdot\vec{b} \tag{2.5}$$

which I prefer. We note here that Phipps[6] points out that "time-dilation" and "mass-increase" for time-like intervals connected to a single particle have ample empirical confirmation, but that "length contraction" has no corresponding *direct* empirical evidence to support it. Clearly evidence against the "Lorentz contraction" of rigid rods would prevent us from using the facile $\sigma^2 = -\tau^2$ assumption we employ in this paper.

Greider remarks that "The four basis vectors $\mathbf{e}_\mu$ are part of the 16 linearly independent elements that form the (Dirac) C_4 algebra, and the v^μ are scalar coefficients. The other 12 elements of C_4 are obtained by multiplication of the $\mathbf{e}_\mu$;..."

In the past I have sometimes simplified my notation $\mathbf{a}(S)$ for a bit-string of length S by dropping the dependence on S. Since this could create confusion with Greider's notation for a 4-vector, I will try not to do so in what follows. In order to distinguish Greider's *space-like* metric (Eq. 2.3, 2.4) from the standard notation in momentum space for *on-shell* 4-vectors p in momentum space[7] we write

$$p := (p^0, p^1, p^2, p^3) = (E, \vec{p});\ p^2 = E^2 - \vec{p}\cdot\vec{p} = m^2$$

Although our strategy for mapping bit-strings onto quaternions works in a formal sense, we do not in this way succeed in achieving Poincar/*acutee* invariance or vector addition. As we will show in the next chapter, bit-string operations do suffice to describe finite and discrete rotations and boosts using strings of fixed length. Further, bit-string concatenation allows us to define multiplication of a single bit-string by positive and negative integers and their combinations, including the scalar "0". This makes our coordinate desciption meaningful, provided we can supply a *macroscopic* ("laboratory") definition of the *directions* of the vector basis strings. I believe this will suffice for the physics modeling I have done and intend to do. I suspect that my failure to construct the full vector addition in our theory has deep roots, but these cannot be explored in this paper.

2.2. Bit-Strings

We specify a *bit-string*

$$\mathbf{a}(S) := (b_1^a b_2^a ... b_s^a .... b_S^a) \tag{2.6}$$

by its S ordered elements

$$b_s^a \in 0,1; \quad s \in 1,2,....S; \quad 0,1,...,S \in \ \textit{ordinal integers} \tag{2.7}$$

and its norm by

$$|\mathbf{a}(S)| := a(S) := \Sigma_{s=1}^{S} b_s^a \tag{2.8}$$

This is the usual Hamming measure for bit-strings. Define the *null string* by $\mathbf{0}(S)$, $b_s^0 := 0$ for all s and the *anti-null* string by $\mathbf{1}(S)$, $b_s^1 := 1$ for all s.

Define *discrimination* by

$$b_s^{a\oplus b} := (b_s^a - b_s^b)^2; \ \mathbf{a}(S) \oplus \mathbf{b}(S) := (....b_s^{a\oplus b} ....b_S^{a\oplus b}) = \mathbf{b}(S) \oplus \mathbf{a}(S) \tag{2.9}$$

Note that this differs from the standard definition of *symmetric difference*, $+_2$, XOR, OREX,... in that our symbols "0", "1" are already specified to be *ordinal* *integers* in some system with maximal ordinal N_X specified in advance, rather than "existence symbols" or "bits". For ours, or for the standard definition, it follows that

$$\mathbf{a}(S) \oplus \mathbf{a}(S) = \mathbf{0}(S);\ \mathbf{a}(S) \oplus \mathbf{0}(S) = \mathbf{a}(S) \tag{2.10}$$

Define $\bar{\mathbf{a}}(S)$ by

$$\bar{\mathbf{a}}(S) := \mathbf{a}(S) \oplus \mathbf{1}(S);\ \textit{hence}\ \mathbf{a}(S) \oplus \bar{\mathbf{a}}(S) \oplus \mathbf{1}(S) = \mathbf{0}(S) \tag{2.11}$$

As noted above, when we employ Greider's bold-face notation for 4-vectors, it is important *always* to include the string length S. It is also important when the norm *and* the anti-null string are involved. In particular

$$|\mathbf{1}(S)| = S;\ |\bar{\mathbf{a}}(S)| = S - a(S) \tag{2.12}$$

If n strings combined by discrimination in all the possible ways taking them $1, 2, 3, ..., n$ at a time do *not* produce the null string, they are said to be *discriminately independent*, or *d.i.* If these combinations also never yield the anti-null string, they are said to be **d**i*scriminately and* **a**nti-**d**i*scriminately independent*, or *d.i.a.d.*

For two strings $\mathbf{a}(S_a), \mathbf{b}(S_b)$ we define *concatenation* ($\|$) by

$$b_k^{a\|b} := b_i^a,\ i \in 1, 2, ..., S_a;\ b_k^{a\|b} := b_j^b,\ j \in 1, 2, ..., S_b, k = S_a + j$$

$$\mathbf{a}(S_a)\|\mathbf{b}(S_b) := (......b_k^{a\|b}.....b_{S_a+S_b}^{a\|b}) \tag{2.13}$$

$$= (...b_i....b_{S_a})\|(....b_j^b...b_{S_b})$$

Hence

$$a(S_a) + b(S_b) = |\mathbf{a}(S_a)\|\mathbf{b}(S_b)| = |\mathbf{b}(S_b)\|\mathbf{a}(S_a)| \tag{2.14}$$

but note that in general $\mathbf{a}(S_a)\|\mathbf{b}(S_b) \neq \mathbf{b}(S_b)\|\mathbf{a}(S_a)$.

2.3. Amson invariance

My tactical motivation for mapping bit-strings onto quaternions is explained at the start of Sec. 2.1; basically, Greider's approach to the free field equations provides us with a familiar point of departure, which we can qualify as we go along. My earlier philosophical motivation for mapping bit-strings onto quaternions came primarily from Amson[8] invariance. This started long ago when I found it useful to obtain "antiparticles" by discriminating with the anti-null string.

It is often emphasized in discussions of bit-strings that so long as the two symbols used in the ordered string are distinct, the choice of what symbols to use is *arbitrary.* Hence there is a basic symmetry in the representational starting point of a theory modeled using bit-strings. John [Amson] emphasized this fact by raising the basic question of where these two symbols come from in the first place. His answer was the "Bi-Orobourous", which is supposed to make them self-contained.

If we define "discrimination" by

$$a \oplus a := 0 = b \oplus b; \ a \oplus b := 1 = b \oplus a$$

where a and b are the two arbitrary, *distinct* symbols already mentioned, it is clear that the two additional symbols "0" and "1" are *also* arbitrary. Using them to replace a and b in a system whose notation is still fluid can be dangerous. If I understand John [Amson] correctly, keeping one pair fixed and interchanging the other pair changes one system into its "dual" system. Then, if I am still on track, this basic symmetry can be collapsed by taking either the a and b or the 0 and 1 as the completed hierarchy in one representation and its dual representation as the initial arbitrary, distinct symbols and starting all over again.

Once we have collapsed the notation by replacing a and b by 0 and 1, we obtain the usual XOR of computer practice in which the symmetry between the two symbols is *broken*, in that the "0" in the definition refers to the symbols being "the same" and the "1" to their being "different". I suspect that this asymmetry is related to Parker-Rhodes' starting point in *Agnosia*[9] and *The Inevitable Universe*[10], where he distinguishes between the ontological statement "something exists" and the information-theoretic statement, "this ontological statement conveys no information".

I make my definition of discrimination still more concrete by defining bit-strings as strings of dichotomous symbols *ordered* by the ordinal integers. I take the normal arithmetic properties of the integers — both with regard to addition and to multiplication — as "given" up to some integer *fixed in advance.* By identifying the dichotomous symbols in the strings — the "0" and "1" — as *ordinal* integers, I make what I claim to be a consistent step, *provided* I define *discrimination* by

$$b_s^{a\oplus b} := (b_s^a - b_s^b)^2;\ b_s^x \in 0,1;\ s \in 1,2,3,....,S$$

rather than using the "symmetric difference " definition given above, or some binary equivalent. This possibility was, like many other things, one that Clive [Kilmister] and I ran into together when working in his office at King's nearly a decade ago. I reiterate here my contention that I see no need for deriving the integers from a more primitive starting point so long as my aim is to model the *practice* of physics in such a way as to *construct* a consistent finite and discrete relativistic quantum mechanics. The philosophical point I wish to make about either my approach, or John [Amson]'s, or Fredrick [Parker-Rhodes]'s, or (so far as I can see) Clive [Kilmister]'s and Ted [Bastin]'s, is that there is a tension between the *broken symmetry* that is an inevitable part of the hierarchy construction as usually presented and the initial indistinguishable duality. I find this contrast fruitful rather than paradoxical.

One has a choice here. The asymmetric structure clearly has a great deal to do with the hierarchical ordering of the scale constants. I claim to have gone a considerable ways toward using this structure to interpret the elementary particle quantum numbers, coupling constants and mass ratios. However, conventional elementary particle physics cannot be formulated without ending up with a theory in which $\mathcal{CPT}$ is necessarily unbroken, even though $\mathcal{C}$,$\mathcal{P}$, $\mathcal{CP}$ (and hence presumably $\mathcal{T}$) are broken *both* empirically *and* in the standard model of quarks and leptons. This was my motivation for invoking "Amson Invariance" a long time ago as the symmetry in our theory which allows us to model this empirical situation. Early on I used discrimination with the anti-null string to distinguish "particles" from "anti-particles". In the current paper I show that my definition of coordinates provides *all* these discrete symmetries. I am working out the details of how this relates, quantitatively, to the way the coupling constants break these symmetries in a manner consistent with experiment.

One important aspect of the theory as I am formulating it is that one has the choice between either breaking $\mathcal{CPT}$ or requiring it. This already gives our approach a critical advantage over conventional theories. A colleague of mine (Helen Quinn) asserts "All relativistic quantum field that anyone has written down are Lagrangian field theories." Further, a standard textbook by Itzykson and Zuber[11] states that "In any quantum field theory derived from a Lagrangian, the PCT theorem holds"; they provide a proof and references to the literature. Max Dresden informs me that the theorem applies only to *local* Lagrangian theories, and that non-local theories have more freedom. Non-local theories would, necessarily, introduce a dimensional parameter for which there is no current experimental motivation. In contrast, the hierarchy construction necessarily breaks $\mathcal{CPT}$ symmetry in any application along the lines I have pioneered; the breaking parameter is part of the theory, i.e. one part in $2^{127} + 136$.

Hamming measure (number of 1's in a string) necessarily breaks "Amson invariance". This fact motivates dropping Hamming measure in favor of a symmetric definition by the way we introduce metric coordinates (see below). In terms of McGoveran's definition of *attribute distance*, what we do is to use some string with an equal number of 0's and 1's as our *reference ensemble*. (Hamming measure uses the null string as the reference ensemble.) This restricts us to using basis strings of even length. We find this to be a good move, because it gives us a simple way to discuss $\mathcal{CPT}$ invariance (see below).

2.4. Orthogonal Bit-String Basis Vectors

In order to give meaning to a vector basis for vectors with integer coefficients constructed from bit-strings we start with a set of D d.i.a.d. *vector basis strings* of the same length S which we call $\mathbf{B}_\alpha(S)$; $\alpha \in 1,2,....,D$. Note that, in contrast with the d.i. basis strings used in the construction of the four levels of the *combinatorial hierarchy* — which by definition exclude the null string — we exclude the anti-null string as well. This suffices for 3-vectors, but for 4-vectors we adjoin the anti-null string explicitly as one of the basis vectors:

$$\mathbf{B}_0(S) := \mathbf{1}(S) \tag{2.15}$$

Once we have selected a d.i.a.d. basis, our next step in defining bit-string addition is to construct a meaning for multiplying a basis string by a positive integer.

$$2\mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(2S) := \mathbf{B}_\alpha(S)\|\mathbf{B}_\alpha(S) \tag{2.16}$$

and hence by recursion

$$(n+1)\mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(S)\|\mathbf{B}_\alpha(nS) = \mathbf{B}_\alpha(nS)\|\mathbf{B}_\alpha(S) = \mathbf{B}_\alpha(S)(n+1) \tag{2.17}$$

Note that

$$|n\mathbf{B}_\alpha(S)| = |\mathbf{B}_\alpha(nS)| = n|\mathbf{B}(S)| = n\ B_\alpha(S) = B_\alpha(nS) = B_\alpha(S)\ n \tag{2.18}$$

Consequently we have indeed succeeded in defining the multiplication of a basis string by a positive integer.

In order to extend this definition to negative integers and multiplication by zero, We define *addition*, $+$, and *subtraction*,$-$, of a basis strings as follows

$$0 := 0\ \mathbf{B}_\alpha(S) := \mathbf{B}_\alpha(S) + \bar{\mathbf{B}}_\alpha(S) \tag{2.19}$$

Hence, since "–"is to have the usual meaning as the inverse of "+",

$$\mathbf{B}_\alpha(S) = -\bar{\mathbf{B}}_\alpha(S) := -\mathbf{B}_\alpha(-S) \tag{2.20}$$

and by recursive definition similar to Eq. 2.16

$$(m \pm n)\mathbf{B}_\alpha(S) = m\mathbf{B}_\alpha(S) \pm n\mathbf{B}_\alpha(S) = \mathbf{B}_\alpha(mS)\|\mathbf{B}_\alpha(\pm nS)$$

$$= \mathbf{B}_\alpha(\mp mS)\|\mathbf{B}_\alpha(-nS) = -\mathbf{B}_\alpha(-S)(m \pm n),\ \ etc. \tag{2.21}$$

We have already restricted ourselves to a d.i.a.d. basis because of our desire to preserve Amson invariance; this motivation *also* requires us to restrict our vector basis strings to strings of even length. For strings of even length (i.e. $\frac{S}{2} \in$ *positive integer*), we call our *vector basis strings* $\mathbf{E}_\mu(S), \mu \in 0, 1, 2, 3...$ and require that

$$\mathbf{E}_0(S) := \mathbf{1}(S);\ |\mathbf{E}_i(S)| = \frac{S}{2},\ i \in 1, 2, 3... \tag{2.22}$$

Then we can *define* the *components* a^μ of any string of length S by

$$a^\mu := |\bar{\mathbf{a}}(S) \oplus \mathbf{E}_\mu(S)| - S/2 \tag{2.23}$$

from which it follows that

$$(E_\mu)^\mu = \frac{S}{2};\ (E_0)^i = 0 = (E_i)^0,\ i \in 1, 2, 3... \tag{2.24}$$

Thus, any "spacial" vector basis string $\mathbf{E}_i(S)$ can be said to be *orthogonal* to the "temporal" vector basis string $\mathbf{E}_0(S)$. In order to have orthogonal coordinates in a $D + 1$ space, we must obviously require that

$$(E_i)^j = \delta_{ij}\frac{S}{2},\ i, j \in 1, 2, ..., D \tag{2.25}$$

We discuss below how this requirement can be met. Once we have established an orthogonal basis of dimensionality D using strings of length S, we can extend the system to include a larger number of coordinates by the "length multiplication" described at the end of the last section. This is simply an (upward) scale change because once this is applied to all the vector basis strings, it is easy to show that

$$(n\mathbf{a}(S))^{\mu} = (\mathbf{a}(nS))^{\mu} = na^{\mu} \tag{2.26}$$

Our mapping of basis vector strings onto basis vectors can now be written as

$$\frac{2}{S}\mathbf{E}_{\mu}(S) \rightarrow \mathbf{e}_{\mu} \tag{2.27}$$

and for repetitive vector basis strings

$$\frac{2}{nS}\mathbf{E}_{\mu}(nS) \rightarrow \mathbf{e}_{\mu} \tag{2.28}$$

2.5. How Many Dimensions?

McGoveran (FDP, Theorem 13) has shown that any discrete space of D "homogeneous and isotropic" dimensions synchronized by a universal ordering operator can have no more than *three* indefinitely continuable dimensions; three separate out and the others "compactify" after a surprisingly small number of constructive operations. The proof starts from the assumption that we have D independent generators of sequences of two dicotomous symbols. The sequences share a common ordinal integer n which is "0" when the sequences start ("initial synchronization") and which counts the number of symbols which have been added sequentially to each sequence; the basic assumption is that whatever method we use to generate the sequences cannot allow any subset of the $d = 1, 2, ..., D$ generators to be distinguished from any of the rest other than by this arbitrary numbering.

For example, we could run the generators until we had produced sequences all D of which are discriminately independent at a length which we could call N_L, the *label length.* Then which we call $d = 1, 2, 3, ..., D$ is an arbitrary replacement for these generated sequences; this is the way part of *PROGRAM UNIVERSE I* operates (cf DP). In Parker-Rhodes terminology, these D sequences are *indistinguishables* with cardinality "D" and ordinality "1". In our context, this is what we mean by "homogeneous and isotropic dimensions". This allows us to invoke a result proved by Feller[12] for D independently generated Bernoulli sequences (i.e. arbitrary sequences of the symbols 0, 1). Feller proved that the probability that after n synchronized trials all will have accumulated the same number of "1" 's is less than $n^{-\frac{1}{2}(D-1)}$. [The exact expression for this probability is $\frac{1}{2^{nD}}\Sigma_{k=1}^{n}(\frac{n!}{k!(n-k)!})^D$.] Consequently the probability of this criterion being met vanishes like $n^{-\frac{3}{2}}$ for $D = 4$, and increasingly rapidly for higher numbers of independently generated sequences. McGoveran met various objections to this interpretation in Ref. 4, Appendix II. For completeness, I quote the relevant passage here.

"Now regarding the difficulty of giving finite combinatorial meaning to Feller's Theorem vis–a–vis statistically unlikely circumstances. While I cannot avoid the statistical character of the proof, I can remove the problem of combinatorial interpretation. This problem arises because of the way Feller invokes convergence and difference theorems and therefore limit theorems. The asymptotic continuation of the combinatorial terms of the series seems to be essential. However, one need not resort to this method to see the validity of the theorem.

"In particular, suppose that a $3 + n$ space has been generated up to some finite extent. Because of the probabilities involved, the most dense constructible 1–dimensional d–subspace will have a denser sequence of metric points than every constructible 2–dimensional d–subspace, and the most dense 2–dimensional d–subspace denser than every 3–dimensional d–subspace. However, this situation reverses at 4–dimensions so that the most dense $4 + n$–dimensional d–subspaces are now ordered as less dense than every 5+n-dimensional d-subspaces (where n is an element of 0, 1, 2, ...)! This means that every $4 + n$–dimensional d–subspace

is separable into a number of isotropic and homogenous 1, 2, and 3–dimensional d–subspaces, but NOT into isotropic and homogenous 1, 2, 3 and 4–dimensional d–subspaces.

"Again, there might be some (and indeed perhaps a large number) of "exceptional" generators of homogeneous and isotropic m–dimensional d–subspaces with $n > 3$. *The algorithm for this generator would be deterministic. However, it is my claim that no such deterministic algorithm can be correct for other reasons as explained regarding "arbitrariness" and the very definition of ordering operator in Foundations: the complexity of the algorithm for an ordering operator is such that it cannot be given a full interpretation within the generated system.*

"For PU, the generators of our d–space, therefore, are of such complexity that the "next" metric mark cannot be represented in terms of all those generated so far. This precludes the possibility that the generation of the space is deterministic in the way required: namely that we can predict deterministically from the d–space generated so far and the distribution of metric marks where/when the next metric mark will be generated. Every c–dimensional d–space with $n > 3$ *is not algorithmicly extensible within the system. It is therefore subject only to statistical characterization. I realize this is not a formal argument and hope to make it formal in my next major effort: Foundations II.*

"Not long ago I questioned Pierre's reference to "McGoveran's Theorem" regarding there being only three conserved unique quantum numbers (which I take to mean that only three quantum units or parameters are possible for global descriptions and what you mean by Pierre's conservation theorem). I subsequently convinced myself that it was OK, with the fourth number being only a locally usable number. If this fourth number is color, we have "color confinement" and "asymptotic freedom". Conservation is not the issue here. (Indeed I insist that nothing ever gets "conserved" but that similar structures are recursively generated so that a "conserved property" is found to have the same "value" over some causal trajectory—see ANPA 11 paper.)

"The argument is simple. PU generates strings with arbitrary quantum numbers (QN's hereinafter) selected from all those allowed. We can imagine a generation which orders the sets of strings with QN's of each type: a set of strings ordered by spin QN, another by angular momentum, etc. We now synchronize the generators so that a d–space is constructed with a diagonal of n strings, one with each of these QN's and therefore n–dimensions. Feller's Theorem now applies.

"I agree that synchronization is the bridge between combinatorics and geometry — at least that is why and how I have used it."

This theorem has a powerful corrolary in our bit-string coordinate context. Eq. 2.22 identifies the vector basis string $\mathbf{E}_0(S)$ with the *unique* anti-null string. If we identify this with the time direction, then all strings which have the same time coordinate $t = a^0(t)$ have the same Hamming measure

$$|\mathbf{a}(S;t)| = t + \frac{S}{2} \tag{2.29}$$

But strings with the same Hamming measure satisfy the condition required by McGoveran's Theorem in Feller's context (i.e. all have the same number of "1" 's). Consequently, any simultaneous (i.e. same "t") points in our finite and discrete space, when constructed from independently generated, but synchronized, sequences of dichotomic variables of the same length S projected onto a coordinate system with D spacial dimensions have a rapidly diminishing probability of satisfying this "distant simultaneity" criterion for large $t + S/2$ and $D > 3$. The critical $D = 3$ case does allow what we call here *DISTANT simultaneity* to be defined for large (but finite) $t + S/2$.

It is important to realize that this DISTANT simultaneity is *non-local* in the usual quantum mechanical sense when we make the interpretation $a^0 = t, \vec{a} = (a^1 = x, a^2 = y, a^3 = z)$. We intend to prove that the basic "three-vertices" correspond to normal relativistic velocity addition in spite of this non-locality of events. When we make the interpretation $a^0 = E, \vec{a} = \vec{p} = (a^1 = p_x, a^2 = p_y, a^3 = p_z)$, and impose the 4-event criterion that 4 strings discriminate to the null string, this is equivalent to 3-momentum conservation in appropriate contexts. Although relativistic 3-momentum is conserved, there is no guarantee that a 3-vertex is "on-shell" in the sense that $E^2 - p^2 = m^2$ for all the "particles". This fact is the starting point of our finite particle number relativistic scattering theory based on relativistic Faddeev-Yakubovsky equations with exactly unitary (flux-conserving) solutions. The asymmetry between the representational properties of "position" and "momentum" already implied by the "counter paradigm" is the reason why an S-matrix type of approach is natural for us. It is also important to realize that our distant simultaneity is *independent* of any concept of *causal continuity* of the type usually associated with special relativity, unless or until we specify in more detail how the strings are generated. That *program universe*-type generators lead to acausal, supraluminal connectivity without allowing supraluminal signaling has been argued elsewhere[13,14] .

2.6. Rational Quaternions

Our mapping of bit-strings onto an orthogonal coordinate system with spacial dimension D works only for even string length and some set of strings which satisfy Eq. 2.25. Further, if $S/2$ is odd, the indistinguishability condition for $D > 1$ implied by Eq. 2.25 cannot be met because two bit-strings with odd Hamming measure discriminate to a bit-string with even Hamming measure. Consequently the simplest basis system we can use for $D > 1$ must have strings which are multiples of some basis of length four. There are $(S!)/(\frac{S}{2}!)^2 = 6$ candidates for the vector basis strings with $S = 4$, $(n = 1)$, but three of these can be obtained from the other three by discrimination with the anti-null string, and correspond to *finite* and *discrete* rotations or reflections of the basis. One allowed basis in three plus one dimensions which I have been studying for some time is

$$\mathbf{E}_0(nS) := n(1111) = \mathbf{1}(nS)$$

$$\mathbf{E}_1(nS) := n(1010); \;\; \mathbf{E}_2(nS) := n(1001); \;\; \mathbf{E}_3(nS) := n(1100) \qquad (2.30)$$

Since we saw in Sec. 2.5 that we need at most 3+1 dimensions, we will confine ourselves to this system from now on.

In order to conform to his notation, Clive [Kilmister] suggests that we use instead

$$\mathbf{K}_0(nS) := n(1111) = \mathbf{1}(nS)$$

$$\mathbf{K}_1(nS) := n(1001);\ \mathbf{K}_2(nS) := n(0101);\ \mathbf{K}_3(nS) := n(0011) \qquad K2.32$$

Here I have called his suggestion $\mathbf{K}_\mu(nS)$, and used my scalar multiplication notation. The advantage is that we can then write

$$\mathbf{K}_i(nS) = n(i4);\ \mathbf{K}_0(nS) = n(1234)$$

This move looks good; it does not change anything below, so far as I can see.

Discrimination of 1, 2, 3 or 4 basis vectors with the anti-null string correspond to well known *discrete* symmetry operations in 3+1 space-time. We list these:

$\mathbf{E}_0(nS)' = \boldsymbol{\mathcal{T}}\mathbf{E}_0(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_0(nS)$ corresponds to TIME inversion.

$\mathbf{E}_i(nS)' = \boldsymbol{\mathcal{M}}\mathbf{E}_i(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_i(nS)$ corresponds to MIRROR REFLECTION across the jk plane.

$\mathbf{E}_{i,j}(nS)' = \boldsymbol{\mathcal{R}}\mathbf{E}_{i,j}(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_i(nS), \mathbf{E}_j(nS)$ corresponds to ROTATION through 180^o around the k axis in either sense.

$\mathbf{E}_{i,j,k}(nS)' = \boldsymbol{\mathcal{P}}\mathbf{E}_{i,j,k}(nS) := \mathbf{1}(nS) \oplus \mathbf{E}_1(nS), \mathbf{E}_2(nS), \mathbf{E}_3(nS)$ corresponds to SPACE inversion — the PARITY operation.

We emphasize the fact that our construction leads immediately to the *discrete* space-time symmetries $\boldsymbol{\mathcal{P}}, \boldsymbol{\mathcal{T}}$ including the degenerate rotation and reflection options. Once we have discussed particulate *quantum numbers*, it will be easy to extend our discussion to $\boldsymbol{\mathcal{C}}$ and the role $\boldsymbol{\mathcal{CPT}}$ invariance plays in our discrete theory.

To go from here to rational quaternions is immediate. Simply define

$$\mathbf{e}_\mu{}^2 = 1 - \frac{4}{nS}(\mathbf{E}_\mu(nS))^0 \tag{2.31}$$

which insures that our basis vectors satisfy Eq. 2.2. We can now follow Greider by adopting the constraint given by Eq. 2.1; then use the components a^μ given by Eq. 2.23 to define a 4-vector given by Eq. 2.3. If the integers we start with do not provide a fine enough mesh to describe the phenomena we are modeling, we can *rescale* as explained above; if we wish to replace integer coordinates by rational coordinates with a smallest aliquot part $1/N_x$ named in advance we can divide all components by this factor. This measure can be fixed in particle physics in the *context* of anticipated experimental resolution. If we wish to use a time-like rather than a space-like metric, all we need do is change the sign of Eq. 2.31,

$$(\mathbf{e}_\mu{}^2)' = \frac{4}{nS}(\mathbf{E}_\mu(nS))^0 - 1 \tag{2.32}$$

So far as *coordinate description* goes, this completes our mapping of bit-strings onto quaternions.

Having gone this far, a temptation for both physicists and continuum mathematicians is to view this mapping of the bit-string spacial coordinates as an *embedding* in R_3, and of the quaternion coordinates as an *embedding* in the space-time of special relativity. Then *coordinate transformations* could be carried through in a conventional way. But this would cut the umbilical cord connecting the mapping to bit-strings. This can easily be seen by trying to go backward after a coordinate transformation and ask what this corresponds to in terms of bit-strings.[15] So we have to do more work on coordinate transformations in order to discover which can be expressed in terms of bit-string operations and which cannot. This is well worth the effort, since the bit-string generated "space" is much sparser in "points" than pedestrian "discretizations of the continuum" might lead one to expect. This fact could provide us with a start toward understanding in a new way why our theory gives us the limiting velocity of relativity and the non-commutativity of quantum mechanics *without* producing at the same time "self-energies" which go to infinity in physically interesting situations, and like horrors.

3. COORDINATE SYSTEM TRANSFORMATIONS

3.1. THE COUNTER PARADIGM REVISITED

In our discussion of the "Counter Paradigm" in DP, pp 90-91, we noted that *"..., we will have to provide more and more precise definitions of these criteria [relating 3- and 4- vertices at certain TICKs to the space-time volumes of laboratory counters] as the analysis develops."*

Physicists are accustomed to "looseness of fit" between the mathematics (*representational framework*, R), the connection to quantitative laboratory measurement (*rules of correspondence, procedural framework*, P), and the objectives of the process (*epistemological framework*, E), whatever names they use for these three essential ingredients in the modeling of the *practice* of physics. In my view only many recursions through RPE in any order can be expected to yield satisfactory results. This looseness generates considerable criticism from some members of ANPA wherever I start. As a physicist, I have been more comfortable starting with E, roughing out the mathematics R enough to make a first stab at connecting to laboratory practice (*including* the way algebraic formulae and monte-carlo programs are used to compare theoretical predictions with digital laboratory results, i.e. "counter data") P, and then recursing to E to get an estimate of where we are; I can then ask what it might be profitable to scrap before going on. This has landed me in mathematical difficulties, sometimes over my head.

My initial thoughts about how to connect the "counter paradigm"[16] to Stein's "random walk" model[17–19] were *very* naive. Recently I have been trying to come to grips with some of the difficult aspects, — *after* Karmanov (Ref. 15) had suggested that we might be able to go directly from a "Stein-like" model to a discrete version of the 1+1 Dirac Equation *without* going to an infinitesimal step-length "limit".

The naive counter paradigm amounts to saying that when we have two sequential counter firings a distance L apart with time separation T attributed to a single particle of mass m, we can associate the invariant interval $c^2\tau^2 = c^2T^2 - L^2$ between these events with a labeled bit-string. The label, according to rules that I am still developing, specifies the mass. If the string $\mathbf{a}(S;m)$ is of length S and Hamming measure $a(S;m) = |\mathbf{a}(S;m)|$, we take the time to be $T = Sh/mc^2$ and the distance to be $L = [2a(S;m) - S](h/mc)$ it follows that the velocity $V = L/T = \beta c = \frac{2\mathbf{a}(S;m)}{S} - 1$. Since, in practice, we cannot measure the dimensions of a counter to an integral number of Compton wavelengths h/mc and the time resolution of the counters is much coarser than h/mc^2, these constraints define an ensemble of strings and not a single string. Part of the problem my lectors have with my exposition is that my language has often led them to identify a particle with a single labeled string rather than with this context-dependent ensemble. I am so used to employing this type of short-hand in going from model to experimental context and back that I tend to forget how often I need to remind others (and occasionally even myself) how inextricably connected this empirical context is to the model itself, however "mathematical" the representation of the model may look.

Once my model is spelled out this way, it is easy to think of the ensemble of strings as a "random walk", or relativistic *Zitterbewegung*, in which the particle takes a step either along or against a line connecting the two counters, each step executed at the velocity of light, defining a causal trajectory in 1+1 space-time generated by the construction of any particular string as a Bernoulli sequence. This is where the trouble starts. Such a model, in the large number case, would approximate a relativistic diffusion equation and *not* the Schroedinger equation. One can use it to derive the Lorentz transformations, as Stein did initially, by treating the step-length as the uncertainty in position; a rigorous derivation along these lines is given by McGoveran in FDP. But this is still a long cry from quantum mechanics.

Stein attacked this problem in his most recent published paper[20]. He distinguished quantum events from classical coincidences in such a way that the quantum process corresponds to a *single step*, and in this way was able to prove that in his model a Gaussian distribution exhibits the characteristic "wave packet spreading" of quantum mechanics. We convinced John Bell that Stein had in this way constructed the solutions of the Schroedinger equation for Gaussian wave packets, and I am willing to argue that he did indeed derive the 1+1 free particle Schroedinger equation in this way. Feynman and Hibbs[21] get the relativistic Schroedinger and Dirac equations out of a similar model, by taking the counter-intuitive leap of treating the step-length as *imaginary*! We will discuss why that works from our point of view on another occasion[22]. Adequate treatment requires much more care than the naive model we have sketched in this section. A preliminary treatment[23] claimed that I had derived the Feynman imaginary step length prescription, but this claim should be treated with caution.

In my first approach to the Lorentz transformations (DP pp 91-93) for the interval connecting coordinates $(0,0)$ to (z,t) in the forward light cone (in units of h/mc for z and h/mc^2 for t) I used $z = 2a(S) - S$, $t = S$, and asked for a transformation from $(z;t)$ to $(z';t')$ which keeps $\tau^2 = t^2 - z^2 = 4a(S)(S - a(S))$ invariant. This is generated by

$$(t' + z') = \rho(t+z) = \rho\ 2a(S);\ \ (t' - z') = \rho^{-1}(t-z) = \rho^{-1}2(S - a(S)) \quad (3.1)$$

or in terms of Hamming measure and string length by

$$a'(S') = \rho\ a(S);\ \ S' = \gamma_\rho S;\ \gamma_\rho = \frac{1}{2}[\rho + \rho^{-1}] \quad (3.2)$$

The difficulty with this route to the Lorentz transformations, which Karmanov realized (Ref. 15) but I did not, is that if we are to retain connection to bit-string operations $\gamma_\rho S$ must be integral.This places a non-linear restriction on the boost velocities allowed in Lorentz transformations. No physical experience calls for this inhomogeneity, so this early approach (unfortunately now enshrined in DP) must be firmly *rejected.*

3.2. WORK IN PROGRESS

The discussions which grew out of Karmanov's proposal to derive the Dirac equation starting from a "Stein-like" interpretation (Ref. 15) made it clear that in order to model the Dirac equation with finite step lengths and *not* end up with a classical diffusion equation, it is necessary to consider two *independently generated* bit-strings. Much of this discussion is reproduced in my contribution to ANPA WEST 6 (Ref. 23) and leads to correct conclusions.

One basic reason why we use two independently generated sequences to discuss the 1+1 Lorentz transformation is that this enables us to keep the string length fixed. In the laboratory this corresponds to keeping the distance between the counters fixed, and hence is expressible in terms of some fixed number of invariant lengths h/mc. In contrast to the situation in classical special relativity, this provides us with a convenient specification of what we mean by a "rigid rod". We hope to show in another paper that this will enable us to *define* "mass ratios" in terms of relativistic de Broglie wave interference, and to *derive* the relativistic version of Mach's definition (3-momentum conservation, or Newton's Third Law) as a consequence. We note that defining mass in terms of length measurement provides an alternative to Wheeler's gravitationally based "geometrodynamics"[24] which could have important conceptual consequences.

The quantum mechanical system we wish to model with bit-strings is a free particle of mass m, energy E, momentum p and hence (in units with the limiting velocity $c = 1$) with velocity $\beta := p/E$; further $E^2 - p^2 := m^2$. Relative to a reference center, it has the square of its total angular momentum (in units of $\hbar^2$) given by $\frac{a}{2}(\frac{a}{2}+1)$ and the projection of that angular momentum onto some reference direction (in units of $\hbar$) — the magnetic quantum number called μ_a — given by an integer or half-integer in the range $-\frac{a}{2} \le \mu_a \le +\frac{a}{2}$. Once we have related these integers to our bit-string notation, we expect to go on to show that in this fully discrete context, we can define the appropriate invariants for rotations and boosts. For an appropriate constructive algorithm, the statistics of the 0's and 1's in the string ensembles which meet the correct boundary conditions provide a discrete representation of the solutions of the free particle Dirac and relativistic Schroedinger equations. Elsewhere we will discuss the composition of angular momenta, scattering theory, coupling constants, mass ratios, ... Earlier work, based rigorously on the *ordering operator calculus* (FDP) but pretty heuristic when it came to physical application (DP, et. seq.) is consistent with the new development being worked out here.

In ordinary one particle quantum mechanics, the space-time reference framework is assumed understood as the normal classical continuum of special relativity. Here we cannot afford that luxury. Instead we start with two bit-strings, a reference string $\mathbf{R}(S)$ and the string of interest $\mathbf{a}(S)$ which are subject to the constraints

$$S > R := R(S) > a := a(S) > 0 \tag{3.3}$$

and select a third integer or half-integer parameter μ_a which lies in the range

$$-\frac{a}{2} \leq \mu_a \leq +\frac{a}{2} \tag{3.4}$$

This is related to the two strings by adopting a standard representation for them:

$$\mathbf{R}(a,\mu_a) = \mathbf{1}(\frac{a}{2}-\mu_a-a)\|\mathbf{0}(\frac{a}{2}+\mu_a)\|\mathbf{1}(R-\frac{a}{2}+\mu_a)\|\mathbf{0}(n_0)$$

$$\mathbf{a}(R;a,\mu_a) = \mathbf{1}(\frac{a}{2}-\mu_a-a)\|\mathbf{1}(\frac{a}{2}+\mu_a)\|\mathbf{0}(R-\frac{a}{2}+\mu_a)\|\mathbf{0}(n_0) \tag{3.5}$$

$$\mathbf{R}(a,\mu_a)\oplus\mathbf{a}(R;a,\mu_a) = \mathbf{0}(\frac{a}{2}-\mu_a-a)\|\mathbf{1}(\frac{a}{2}+\mu_a)\|\mathbf{1}(R-\frac{a}{2}+\mu_a)\|\mathbf{0}(n_0)$$

Where

$$n_0 = S-[R+\frac{a}{2}+\mu_a] > 0$$

determines the string length. Note that this parameter is *arbitrary* so long as the other conditions are met. Further, so long as the *same* permutation of the positions

$s \in 1, 2, 3, ..., S$ is applied to all three strings, the properties of interest for this minimal structure are unchanged. It is the existence of these $S!$ permutations that lead to a different count for our probabilities than one would obtain by thinking of the strings as Bernoulli sequences (see Ref. 4, esp. Appendix III). We expect to see in due course that this arbitrariness in the string length can replace the accepted arbitrariness of the phase parameter in the quantum mechanical wave function. We expect to show that the dependence on string length in our theory will be negligible for large enough string lengths in the physical situations currently accessible technologically.

....

Define

$$[A_-A_+](a, \mu_a) := (\frac{a}{2} + \mu_a)(\frac{a}{2} + 1 - \mu_a) = \frac{a}{2}(\frac{a}{2} + 1) + \mu_a - \mu_a^2$$

$$[A_+A_-](a, \mu_a) := (\frac{a}{2} - \mu_a)(\frac{a}{2} - 1 + \mu_a) = \frac{a}{2}(\frac{a}{2} + 1) - \mu_a - \mu_a^2 \qquad (3.6)$$

Hence

$$A_\perp^2 := \frac{1}{2}[A_-A_+ + A_+A_-](a, \mu_a) = \frac{a}{2}(\frac{a}{2} + 1) - \mu_a^2 \qquad (3.7)$$

GOOD HUNTING!

Much could be accomplished by working out the implications of this definition.

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